Bonus Assignment

Grading. The bonus assignment has a maximum of 6 points. The total number of points earned will be added on to your grade received on the midterm on which you received the lowest mark. That is if your lowest midterm mark was 30 points (out of 50) it will become 36 if you get full marks on the bonus assignment.

Software. This assignment is intended to be done using either Macaulay2 (`http://www.math.uiuc.edu/Macaulay2/`) or Sage (`http://www.sagemath.org/`). Submissions done using other software packages will not be marked. Both Macaulay2 and Sage may be freely downloaded and installed on your laptop, or used online at `http://habanero.math.cornell.edu:3690/` for Macaulay2 and `http://sagecell.sagemath.org/` or `https://cloud.sagemath.com/` for Sage. In the questions below "CAS" will refer to your chosen Computer Algebra System, i.e. either Macaulay2 or Sage. To receive marks for this assignment **you must include in your submitted assignment which Macaulay2 or Sage commands you used to answer the questions**.

Problem 1. We know that all nonzero prime ideals are maximal in a principal ideal domain. This means if F is a field and I is a prime ideal in $F[x]$ then I is also a maximal ideal in $F[x]$. Do the following things in your CAS:

- Show that the quotient ring
$$S = \mathbb{Z}_{977}[x]/\left\langle 7x^{11} + 4x^5 - 23x^4 + x - 27\right\rangle$$
is a field.
- Find the multiplicative inverse of $x^2 - 1$.
- Verify that your answer is indeed the multiplicative inverse of $x^2 - 1$ in S.

Problem 2. In your CAS define the polynomial ring $\mathbb{Q}[z]$ and the ideals
$$I = \left\langle z^5 - 14z^4 + 49z^3 - 13z^2 - 63z + 28, z^3 - 16z^2 + 67z - 28\right\rangle$$
and $J = \langle z^3 - 7z^2 + 57, z^5 + 34z - 12\rangle$. Use your CAS to perform/answer the following:

- Find a principal generator of the ideal I, that is find a polynomial $f(z) \in \mathbb{Q}[z]$ such that $I = \langle f(z)\rangle$.
- Is J a proper ideal of $\mathbb{Q}[z]$?

Problem 3. Let $S = \mathbb{Q}[y]/\langle y^4 - 7y^2 + 12\rangle$; use your CAS to:

- Show that S is not an integral domain.
- Find a pair of zero divisors in S.